PHYSICAL REVIEW VOLUME 134, NUMBER 4B 25 MAY 1964

Pseudoscalar Charge Density of Spin-$\frac{1}{2}$ Particles. II. Observability*

K. HIIDA†
Argonne National Laboratory, Argonne, Illinois
(Received 5 December 1963; revised manuscript received 2 February 1964)

To show clearly that the pseudoscalar charge density is a physical observable, the influence of the pseudoscalar charge density on the spin orientation of a charged spin-$\frac{1}{2}$ particle in time-independent electric and magnetic fields is considered. The following results are found: (I). *Longitudinal electric field*. Because of the interaction of the pseudoscalar charge density with the field, the spin rotates about the axis perpendicular to the direction of propagation and the spin orientation. (II). *Transverse electric field*. The interaction of the pseudoscalar charge density with the field causes the spin to rotate about the axis perpendicular to the spin and the direction of the field. The interaction of the usual scalar charge density with the field causes the spin to rotate about the axis perpendicular to the direction of propagation and the field. At very low energies, the spin can be rotated through a larger angle in the former case than in the latter. (III). *Longitudinal and transverse magnetic fields*. There is no interaction between the induced pseudovector current and the magnetic fields.

WHEN interactions are renormalizable and are invariant under space reflection and charge conjugation (CP invariant), then under the requirements that the S matrix is free from divergences after the renormalization and that it is gauge invariant, it was shown in a preceding paper[1] that for any spin-$\frac{1}{2}$ particle with nonvanishing mass, the pseudoscalar charge density should be induced by parity-nonconserving interactions. Unrenormalizable weak interactions would be a source of the pseudoscalar charge density. The photon vertex of any spin-$\frac{1}{2}$ particle with nonvanishing mass on the mass shell is expressed [Eq. (3.4) of II] as

$$\begin{aligned}\bar{\psi}(p_1)j_\mu(p_1,p_2)\psi(p_2)&\\ =ie\bar{\psi}(p_1)\{&\Gamma_\mu F_1(q^2)+(1/2m)\sigma_{\mu\nu}q_\nu F_2(q^2)\\ &+(1/m^2)[q^2\Gamma_\mu-(\Gamma\cdot q)q_\mu]\gamma_5F_3(q^2)\}\psi(p_2)\,, \quad (1)\end{aligned}$$

where

$$q=(p_1-p_2)\,,\quad \sigma_{\mu\nu}=(i/2)(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\,,$$
$$\Gamma_\mu=(1-a^2)^{-\frac{1}{2}}\gamma_\mu(1+a\gamma_5)\,,$$

and two gamma matrices γ_μ and Γ_μ satisfy the same commutation relation: $\{\gamma_\mu,\gamma_\nu\}=\{\Gamma_\mu,\Gamma_\nu\}=2\delta_{\mu\nu}$. Under the requirement that the wave-function renormalization constant z_2 of the particle should have the meaning of a probability, it was shown in a previous work[2] that the upper limit of the constant a^2 is $a^2\leqslant\frac{1}{3}$. (See also the note added in proof.) The factor $a(1-a^2)^{-\frac{1}{2}}\psi^*(p_1)\gamma_5\psi(p_2)F_1(q^2)$ in the first term is the induced pseudoscalar charge density. The factor $a(1-a^2)^{-\frac{1}{2}}\bar{\psi}(p_1)\gamma\gamma_5\psi(p_2)F_1(q^2)$ in the same term is the induced pseudovector current density. The second and third terms in Eq. (1) are the anomalous-magnetic-moment and the anapole-moment terms,[3] respectively.

It was argued in the last section of II that the pseudoscalar charge density may be, in principle, a physical observable. The purpose of this work is to present an example showing clearly that the pseudoscalar charge density is a physical observable. To show this, we shall start from the Lagrangian density of a free spin-$\frac{1}{2}$ particle [Eq. (8) of I] which is given by

$$L_0=-\bar{\psi}(x)[\Gamma_\mu(\partial/\partial x_\mu)+m]\psi(x)\,. \qquad (2)$$

This leads to the spin vector

$$\mathbf{M}=\tfrac{1}{2}\psi^*(x)S^{-2}\boldsymbol{\sigma}\psi(x)\,, \qquad (3)$$

where $\boldsymbol{\sigma}$ is the vector of the 2×2 Pauli spin matrix, and [Eq. (4.2) of II]

$$S=\left\{\frac{(1-a^2)^{1/2}}{2[1+(1-a^2)^{1/2}]}\right\}^{1/2}\left\{1+\frac{(1-a\gamma_5)}{(1-a^2)^{1/2}}\right\}.$$

Substituting the positive-energy solution of the free particle at rest for ψ in Eq. (3) leads to[4]

$$\begin{aligned}2M_x&=A^*B+AB^*\,,\\ 2M_y&=-iA^*B+iAB^*\,,\\ 2M_z&=A^*A-B^*B\,,\end{aligned} \qquad (4)$$

where A and B denote the first and second components of ψ, respectively. This expression shows that the spin orientation of the free particle at rest is determined only by A and B, and that a has no physical meaning for the free particle.

* Work performed under the auspices of the U. S. Atomic Energy Commission while on leave of absence from the Research Institute for Fundamental Physics, Kyoto University, Kyoto, Japan.

† Present address: Institute for Nuclear Study, University of Tokyo, Tanashi-Mach, Kitatama-Gun, Tokyo, Japan.

[1] K. Hiida, Phys. Rev. **134**, B174 (1964). This will be cited hereafter as II.

[2] K. Hiida, Phys. Rev. **132**, 1239 (1963). This will be cited hereafter as I.

[3] Ya. B. Zel'dovich, Zh. Eksperim. i Teor. Fiz. **33**, 1531 (1957) [English transl.: Soviet Phys.—JETP **6**, 1184 (1958)].

[4] In this paper we shall use the following notations and representations:

$$\boldsymbol{\alpha}=i\beta\boldsymbol{\gamma}=\begin{pmatrix}0&\boldsymbol{\sigma}\\ \boldsymbol{\sigma}&0\end{pmatrix},\quad \beta=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad \gamma_5=\begin{pmatrix}0&-1\\-1&0\end{pmatrix},$$

$$\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad\text{and}\quad \sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

As is well known, the infinitesimal operator $[1-\frac{1}{2}i\sigma_i\delta\theta]$ operating on $\begin{pmatrix}A\\B\end{pmatrix}$ rotates $\mathbf{M}$ by angle $\delta\theta$ about the i axis. When parity is not conserved in interactions, the type of infinitesimal operator $[1+\frac{1}{2}(M_i/M)\delta\theta-\frac{1}{2}\sigma_i\delta\theta]$ appears in the theory, where $M=|\mathbf{M}|$, $M_i=\mathbf{i}\cdot\mathbf{M}$, and $\mathbf{i}$ is the unit vector in the i direction. By use of the definition of the spin orientation, Eq. (4), it is shown that the infinitesimal operator $[1+\frac{1}{2}(M_i/M)\delta\theta-\frac{1}{2}\sigma_i\delta\theta]$ operating on $\begin{pmatrix}A\\B\end{pmatrix}$ rotates the spin orientation $\mathbf{M}$ about the axis $(\mathbf{i}\times\mathbf{M})$ by the angle

$$\delta\rho=\delta\theta\sin\rho\,,\tag{5}$$

where $\cos\rho=M_i/M$. The spin orientation of the state $\begin{pmatrix}A\\B\end{pmatrix}$ and the transformed state $\begin{pmatrix}A'\\B'\end{pmatrix}\equiv[1+\frac{1}{2}(M_z/M)\delta\theta-\frac{1}{2}\sigma_z\delta\theta]\begin{pmatrix}A\\B\end{pmatrix}$ are denoted by $\mathbf{M}$ and $\mathbf{M}'$, respectively. Then Eq. (4) leads to

$$\begin{aligned}M_x'&=[1+(M_z/M)\delta\theta]M_x\,,\\ M_y'&=[1+(M_z/M)\delta\theta]M_y\,,\\ M_z'&=[1+(M_z/M)\delta\theta]M_z-M\delta\theta\,,\\ M'&=M\,,\end{aligned}$$

where $M'=|\mathbf{M}'|$ and the terms of the square and higher orders of $\delta\theta$ are neglected. The last equation, $M'=M$, means that the operator $[1+\frac{1}{2}(M_z/M)\delta\theta-\frac{1}{2}\sigma_z\delta\theta]$ does not change the length of the vector $\mathbf{M}$ by the transformation. The remaining three equations show that the operator rotates the vector $\mathbf{M}$ about the axis $(\mathbf{k}\times\mathbf{M})$ by the angle

$$\left\{\left[\frac{M_z'}{M'}-\frac{M_z}{M}\right]^2+\left[\frac{(M_x'^2+M_y'^2)^{1/2}}{M'}-\frac{(M_x^2+M_y^2)^{1/2}}{M}\right]^2\right\}^{1/2}=\frac{(M_x^2+M_y^2)^{1/2}}{M}\delta\theta=\delta\theta\sin\rho\,,$$

where $\mathbf{k}$ is the unit vector in the z direction and $\cos\rho=M_z/M$.

We shall calculate the influence of the pseudoscalar charge density on the spin orientation of a charged spin-$\frac{1}{2}$ particle in an external electromagnetic field. By taking the form factors appearing in Eq. (1) as $F_1(q^2)\equiv1$ and $F_2(q^2)=F_3(q^2)\equiv0$, the problem will be simplified. Then the equation of motion for the charged particle is

$$\left[\Gamma_\mu\left(\frac{\partial}{\partial x_\mu}-ieA_\mu\right)+m\right]\psi(x)=0\,.\tag{6}$$

Let us consider the steady-state problem, that is, the case in which the external field $A_\mu=(\mathbf{A},i\Phi)$ is time-independent. Then the wave function ψ has the time dependence $\exp[-iWt]$, where $W^2=\mathbf{p}^2+m^2$ and $\mathbf{p}$ is the momentum of the particle before it reaches the region where the external field exists. The four-component equation (6) then reduces to two component equations:

$$\begin{aligned}\Bigg\{\pi_0^2-m^2-\boldsymbol{\pi}\cdot\boldsymbol{\pi}+e\boldsymbol{\sigma}\cdot\mathbf{H}-\frac{ie(1-a^2)^{1/2}}{[m+(1-a^2)^{1/2}\pi_0]}\mathbf{E}\cdot\boldsymbol{\pi}\\ +\frac{e(1-a^2)^{1/2}}{[m+(1-a^2)^{1/2}\pi_0]}\boldsymbol{\sigma}\cdot(\mathbf{E}\times\boldsymbol{\pi})\\ -\frac{iaem}{[m+(1-a^2)^{1/2}\pi_0]}\boldsymbol{\sigma}\cdot\mathbf{E}\Bigg\}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}=0\,,\end{aligned}\tag{7}$$

and

$$\begin{pmatrix}\psi_3\\\psi_4\end{pmatrix}=\frac{1}{[m+(1-a^2)^{1/2}\pi_0]}[(1-a^2)^{1/2}\boldsymbol{\sigma}\cdot\boldsymbol{\pi}+am]\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix},$$

where

$$\psi=\begin{bmatrix}\psi_1\\\psi_2\\\psi_3\\\psi_4\end{bmatrix},\quad\boldsymbol{\pi}=(-i\boldsymbol{\nabla}-e\mathbf{A})\,,\quad\pi_0=(W-e\Phi)\,,$$

and E and H are the strengths of the external electric and magnetic fields, respectively, defined by

$$\boldsymbol{\nabla}\Phi=-\mathbf{E}\quad\text{and}\quad\mathbf{H}=\boldsymbol{\nabla}\times\mathbf{A}\,.$$

In the equation of motion (7), the last term shows the interaction of the pseudoscalar charge density $ae(1-a^2)^{-\frac{1}{2}}\psi^*\gamma_5\psi$ with the external field Φ. As will be shown, the existence of this term means that the solution of Eq. (7) includes the spin-rotation operator of the form $[1+\frac{1}{2}(M_i/M)-\frac{1}{2}\sigma_i\delta\theta]$. The second and the third terms from the last in Eq. (7) show that interaction of the scalar charge density $e(1-a^2)^{-\frac{1}{2}}\psi^*\psi$ with the external field Φ. These terms also depend on the constant a^2. As is well known,[5] the terms including the spin dependence of $\boldsymbol{\sigma}\cdot\mathbf{H}$ and $\boldsymbol{\sigma}\cdot(\mathbf{E}\times\boldsymbol{\pi})$ contribute to the spin-rotation operators of the form $[1-\frac{1}{2}i\sigma_i\delta\theta]$. As was shown in Sec. 5 of I and in Eq. (7), in our theory there appear the terms $ae\boldsymbol{\sigma}\cdot\mathbf{E}$ and $(ae\phi)^2$ which show the interaction of the pseudoscalar charge density with the external field Φ, but the terms $ae\boldsymbol{\sigma}\cdot\mathbf{H}$ and $(ae)^2\mathbf{A}\cdot\mathbf{A}$ do not appear. The latter terms show the interaction of the induced pseudovector current $ae(1-a^2)^{-\frac{1}{2}}\bar{\psi}\gamma\gamma_5\psi$ with the external field $\mathbf{A}$. This is the reason that the title of this paper is "Pseudoscalar Charge Density" rather than "Pseudovector Current Density." When only the magnetic field $\mathbf{H}$ exists, therefore, there is no difference between the present theory and the usual one.

To show that the pseudoscalar charge density is a physical observable, let us estimate the effect of the last term in Eq. (7) on the spin orientation of the charged particle in longitudinal and transverse electric

[5] H. A. Tolhoek and S. R. De Groot, Physica **17**, 17 (1951).

fields. We shall consider the case in which electric fields exist only in the infinitesimal region $0\leqslant x\leqslant x'$, and are constant in space and time.

A. LONGITUDINAL ELECTRIC FIELD

The case in which the direction of the electric field is parallel to the direction in which the particle was propagated at $x<0$, the equation of motion (7) reduces to

$$\left\{W^2-m^2+\frac{\partial^2}{\partial x^2}+2eEWx+e^2E^2x^2 -\frac{(1-a^2)^{1/2}aE}{\{m+(1-a^2)^{1/2}[W+eEx]\}}\frac{\partial}{\partial x} -\frac{iaeEm}{\{m+(1-a^2)^{1/2}[W+eEx]\}}\sigma_x\right\}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=0, \quad (8)$$

where $\mathbf{E}=E\mathbf{i}$ and $\mathbf{i}$ is the unit vector in the x direction. We look for the solution of Eq. (8) of the form[5]

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\substack{t=t\\ x=x'}}=\exp[ief]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\substack{t=0\\ x=0}}\exp[ipx'-iWt],$$

where

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\substack{t=0\\ x=0}}=\begin{pmatrix}A\\ B\end{pmatrix}.$$

The solution is

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\substack{t=t\\ x=x'}}=\left(1+\frac{1}{2}\frac{M_x}{M}\delta\theta-\tfrac{1}{2}\sigma_x\delta\theta\right)\begin{pmatrix}A\\ B\end{pmatrix} \times\exp\left\{ipx'-iWt+\frac{ieEW}{2p}(x')^2\right\}\times\exp\left\{-\frac{eEW}{2p^2}x' +\frac{(1-a^2)^{1/2}eE}{2[m+(1-a^2)^{1/2}W]}x'-\frac{1}{2}\frac{M_x}{M}\delta\theta\right\}, \quad (9)$$

where

$$\frac{M_x}{M}=\frac{A^*B+AB^*}{A^*A+B^*B},$$

$$\delta\theta=-\frac{aeEm}{p[m+(1-a^2)^{1/2}W]}x'.$$

The first factor in the expression (9) comes from the interaction of the pseudoscalar charge density with the external electric field and shows the observable effect of the pseudoscalar charge density. The angle between the direction of propagation of the incident beam and that of the spin of the incident beam is denoted by ρ, i.e., $(\mathbf{i}\cdot\mathbf{M})=\cos\rho$. Then the first factor rotates the spin orientation of the incident beam about the axis $(\mathbf{i}\times\mathbf{M})$ by the angle

$$\delta\rho=-\frac{aeEmx'}{p[m+(1-a^2)^{1/2}W]}\sin\rho \quad (10)$$

at infinitesimal positive x'. For the electron and muon, the sign and the magnitude of a was estimated in Sec. 7 of I. It is positive definite and $a\lesssim 10^{-2}$. Further, for them $e=-|e|$. Therefore,

$$\delta\rho>0 \quad \text{for} \quad \pi>\rho>0$$

for the electron and the muon. The positive definiteness of $\delta\rho$ means that the spin orientation of the electron and the muon moving in the direction of the electric field rotates towards the direction opposite to the direction of propagation and the electric field unless $\rho=0$ or π. When the pseudoscalar charge density is zero, $a=0$, the spin vector is not rotated.[5] Thus, the rotation of the spin orientation through the angle $\delta\rho$ given by Eq. (10) in the longitudinal electric field is a remarkable effect of the pseudoscalar charge density.

B. TRANSVERSE ELECTRIC FIELD

The second case to be considered is that in which the direction of an electric field is perpendicular to the direction in which the particle had been propagated at $x<0$. In this case the equation of motion (7) reduces to

$$\left\{W^2-m^2+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+2eEWy+e^2E^2y^2 -\frac{(1-a^2)^{1/2}eE}{[m+(1-a^2)^{1/2}(W+eEy)]}\frac{\partial}{\partial y} +\frac{i(1-a^2)^{1/2}eE}{[m+(1-a^2)^{1/2}(W+eEy)]}\sigma_z\frac{\partial}{\partial x} -\frac{iaeEm}{[m+(1-a^2)^{1/2}(W+eEy)]}\sigma_y\right\}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=0, \quad (11)$$

where $\mathbf{E}=E\mathbf{j}$ and $\mathbf{j}$ is the unit vector in the y direction. The solution of Eq. (11) is

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\substack{t=t\\ x=x'}}=\left(1+\frac{1}{2}\frac{M_y}{M}\delta\theta_y-\tfrac{1}{2}\sigma_y\delta\theta_y\right)\left(1-\frac{i}{2}\sigma_z\delta\theta_z\right)\begin{pmatrix}A\\ B\end{pmatrix} \times\exp\left(ipx'-iWt+\frac{ieEW}{p}x'y\right)\exp\left(-\frac{1}{2}\frac{M_y}{M}\delta\theta_y\right), \quad (12)$$

where

$$\frac{M_y}{M}=\frac{i(AB^*-A^*B)}{A^*A+B^*B},$$

$$\delta\theta_y=-\frac{aeEm}{p[m+(1-a^2)^{1/2}W]}x',$$

$$\delta\theta_z=\frac{(1-a^2)^{1/2}eE}{[m+(1-a^2)^{1/2}W]}x'.$$

The first factor in expression (12) comes from the interaction of the pseudoscalar charge density with the electric field. This factor rotates the spin of the incident particle about the axis $(\mathbf{j}\times\mathbf{M})$ through the angle

$$\delta\rho=-\frac{aeEm}{p[m+(1-a^2)^{1/2}W]}x'\sin\rho,$$

where $(\mathbf{j}\cdot\mathbf{M})=\cos\rho$. The second factor in (12) comes from the interaction of the scalar charge density with the electric field. This factor rotates the spin of the incident beam about the z axis through angle $\delta\theta_z$. At the low-energy limit, the ratio of the two angles $\delta\rho$ and $v\theta_z$ is

$$\delta\rho/\delta\theta_z=-a(1-a^2)^{-\frac{1}{2}}(c/v)\sin\rho,$$

where c and v are the velocity of light and of the incident beam. Thus, there is a possibility at very low energy that $\delta\rho>\delta\theta_z$; that is, as far as the spin rotation is concerned, the effect of the pseudoscalar charge density is larger than the effect of the scalar charge density.

Thus, it has been shown that the pseudoscalar charge density is an observable.

Note added in proof. It was shown in I and II that $a^2\leq\frac{1}{3}$. Here we shall improve the upper limit of a^2. Among the spectral functions ρ_i, the inequality

$$(u^2+v^2+w^2-2uv-2auw+2avw)\rho_1+\frac{2u}{x}(v+aw)\rho_2-\{2vw+a(-u^2+v^2+w^2)\}\rho_3\geq 0$$

holds, where u, v, and w are any real numbers. From this inequality one obtains

$$\begin{aligned}\rho_1\pm\rho_3&\geq 0,\\(2x\rho_1-\rho_2)\rho_2-x^2\rho_3{}^2&\geq 0,\\\rho_2&\geq 0.\end{aligned}\qquad(\alpha)$$

Since $a^2<1$, $\rho_1\pm a\rho_3\geq 0$. From the first equation of (α) and the expressions

$$Z_2{}^{-1}=\int_0^\infty dx^2[\rho_1-a\rho_3]\quad\text{and}\quad\frac{a}{1-a^2}=-Z_2\int_0^\infty dx^2\rho_3,$$

one can prove

$$1>Z_2\geq 0\quad\text{and}\quad a^2\leq\tfrac{1}{4}.$$

ACKNOWLEDGMENTS

The author would like to thank Dr. M. Hamermesh and Dr. M. Peshkin for their warm hospitality at the Argonne National Laboratory and Dr. F. E. Throw for careful reading of the manuscript.

PHYSICAL REVIEW VOLUME 134, NUMBER 4B 25 MAY 1964

Mixing of Elementary Particles

SIDNEY COLEMAN*
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

AND

HOWARD J. SCHNITZER†
Department of Physics, Brandeis University, Waltham, Massachusetts
(Received 3 October 1963; revised manuscript received 12 February 1964)

Particle mixing is studied in a field-theoretic context, as a further approximation to the pole approximation. Although particle mixing is well suited for treating spinless particles, another approximation, also a further approximation to the pole approximation, called vector mixing, is better for treating particles of spin one. Vector mixing is applied to several processes involving the mixing of the ω and the ϕ by the interaction that breaks unitary symmetry.

I. INTRODUCTION

PARTICLE mixing approximations in elementary particle physics have been used by Gell-Mann and Pais[1] (neutral K-meson mixing due to the weak interactions), Glashow[2] (ρ-ω mixing due to electromagnetism), and Okubo[3] (ω-ϕ mixing due to the unknown interaction that breaks unitary symmetry.) All these authors have discussed particle mixing within the framework of a Schrödinger equation acting on the space of one-particle states; the relation of the approximation to the usual approximations of elementary particle physics, derived from field theory or dispersion relations, is by no means clear. It is our intent here to discuss particle mixing within a field-theoretic context, as a further approximation to the pole approximation.

* Work supported in part by the U. S. Air Force Office of Scientific Research Contract No. 49 (638) 589.
† Work supported in part by the National Science Foundation.
[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[2] S. L. Glashow, Phys. Rev. Letters **7**, 469 (1961).
[3] S. Okubo, Phys. Letters **5**, 165 (1963). See also S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963); J. J. Sakurai, Phys. Rev. **132**, 434 (1963); R. Dashen and D. Sharp, Phys. Rev. **133**, 1585 (1964).